INDIAN SCHOOL MUSCAT

NAME OF THE EXAMINATION	SECOND PERIODIC TEST	CLASS: XII
DATE OF EXAMINATION	30 . 05. 22	SUBJECT: MATHEMATICS
TYPE	MARKING SCHEME	SET - A

SET A	Q.NO	VALUE POINTS	MARK
	1.	$y = log\sqrt{\frac{1+sinx}{1-sinx}} = \frac{1}{2}\ log\left(\frac{1+sinx}{1-sinx}\right)$ $= \frac{1}{2}[log(1+sinx) - log(1-sinx)]$ $\therefore \frac{dy}{dx} = \frac{1}{2}\left[\frac{cosx}{1+sinx} - \frac{-cosx}{1-sinx}\right]$ Simplifying to get $\frac{dy}{dx} = secx.$	½ mk ½ mk ½ mk ½ mk
	2.	$sin^2y + cos(xy) = k$ Diff w.r.t x, $2\ siny.cosy.\frac{dy}{dx} - sin(xy)\left(x.\frac{dy}{dx} + y\right) = 0$ Simplifying to get $\frac{dy}{dx} = \frac{y\ sin(xy)}{sin2y - x\ sin(xy)}$	1 mk 1 mk
	3.	$x = 4t, \quad y = \frac{4}{t^2}$ Getting $\frac{dx}{dt} = 4$ and $\frac{dy}{dt} = -\frac{8}{t^3}$ Getting $\frac{dy}{dx} = -\frac{2}{t^3}$ Getting $\frac{d^2y}{dx^2} = \frac{6}{t^4} \times \frac{dt}{dx} = \frac{3}{2t^4}$	½ + ½ ½ mk ½ mk

	4.	$y = log\left(x + \sqrt{1+x^2}\right)$ $\frac{dy}{dx} = \frac{1}{x+\sqrt{1+x^2}}\left(1 + \frac{2x}{2\sqrt{1+x^2}}\right)$ Simplifying to get $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$ Squaring and cross multiplying to get $(1+x^2)\left(\frac{dy}{dx}\right)^2 = 1$ Diff again w.r.t x and dividing by 2 $\frac{dy}{dx}$, we get $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$	1 mk ½ mk ½ mk ½ + ½
	5.	$f(x) = \begin{cases} ax^2+b, & x<1 \\ 2x+1, & x \geq 1 \end{cases}$ is differentiable at $x = 1.$ Given that f is differentiable at $x = 1 \Longrightarrow 2a(1) = 2$ $(LHD = RHD)$ Solving to get $a = 1$ Since f is differentiable, it is continuous. $\big(LHL = RHL = f(1)\big)$ $\Longrightarrow a + b = 3 \quad \therefore b = 2$	1 mk ½ mk 1 mk ½ mk
	6.	$y = x^{logx} + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ Let $u = x^{logx}$ $\Longrightarrow \log u = (logx)^2$ Diff w.r.t x to get $\frac{1}{u}\frac{du}{dx} = 2\ logx.\frac{1}{x}$ $\therefore \frac{du}{dx} = x^{logx}\frac{2\ logx}{x}$ --------- (i) Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ Put $x = tan\theta$ so that $v = \cos^{-1}(\cos 2\theta) = 2\theta$	2 mks 1 ½ mk

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		$\Longrightarrow v = 2\ \tan^{-1} x \quad \therefore \frac{dv}{dx} = \frac{2}{1+x^2}$ ------(ii) Now, $\frac{dy}{dx} = x^{logx} \frac{2\ logx}{x} + \frac{2}{1+x^2}$	½ mk
	7.	(i) To show $g(x) = \lvert x - 10 \rvert$ is not differentiable at $x = 10$ LHD : $\lim_{h\to 0} \frac{f(10-h)-f(10)}{-h} = \lim_{h\to 0} \frac{\lvert 10-h-10\rvert - 0}{-h} = -1$ RHD : $\lim_{h\to 0} \frac{f(10+h)-f(10)}{h} = \lim_{h\to 0} \frac{\lvert 10+h-10\rvert - 0}{h} = 1$ Since LHD≠ RHD, $g(x) = \lvert x - 10 \rvert$ is not differentiable at $x = 10$ (ii) To check the continuity of function $g(x) = \lvert x - 10 \rvert$ at $x = 10$. LHL: $\lim_{x\to 10} (10 - x) = 10 - 10 = 0$ RHL: $\lim_{x\to 10} (x - 10) = 10 - 10 = 0$ And $g(10) = 0$ Since LHL= RHL, $g(x) = \lvert x - 10 \rvert$ is continuous at $x = 10$.	2 mks 2 mks

SET B	Q.NO	VALUE POINTS	MARK
	1.	$tan^2y + cot(xy) = a$ Diff w.r.t x, $2\ tany.sec^2y.\frac{dy}{dx} - xcosec^2(xy)\left(x.\frac{dy}{dx} + y\right) = 0$ Simplifying to get $\frac{dy}{dx} = \frac{y\ cosec^2(xy)}{2\ tany.sec^2y - xcosec^2(xy)}$	1 mk 1 mk
	2.	$x = t^2, \quad y = \frac{4}{t}$ Getting $\frac{dx}{dt} = 2t$ and $\frac{dy}{dt} = -\frac{4}{t^2}$ Getting $\frac{dy}{dx} = -\frac{2}{t^3}$ Getting $\frac{d^2y}{dx^2} = -\frac{6}{t^4} \times \frac{dt}{dx} = \frac{3}{t^5}$	½ + ½ ½ mk ½ mk
	3.	$f(x) = \frac{logx}{x}$ Getting $f'(x) = \frac{1-\log x}{x^2}$ Now, $f'(1) = 1.$	1 mk 1 mk
	4.	$f(x) = \begin{cases} x^2, & x \leq 2 \\ ax + b, & x > 2 \end{cases}$ is differentiable at $x = 2.$ Given that f is differentiable at $x = 2 \Longrightarrow 2(2) = a + 0$ $(LHD = RHD)$ Solving to get $a = 4$ Since f is differentiable, it is continuous $\left(LHL = RHL = f(2)\right)$ $\Longrightarrow (2)^2 = 2a + b \qquad \therefore b = -4$	1 mk ½ mk 1 mk ½ mk

	5.	$y = log\left(x + \sqrt{4 + x^2}\right)$ $\frac{dy}{dx} = \frac{1}{x + \sqrt{4 + x^2}}\left(1 + \frac{2x}{2\sqrt{4 + x^2}}\right)$ Simplifying to get $\frac{dy}{dx} = \frac{1}{\sqrt{4+x^2}}$ Squaring and cross multiplying to get $(4 + x^2)\left(\frac{dy}{dx}\right)^2 = 1$ Diff again w.r.t x and dividing by 2 $\frac{dy}{dx}$, we get $(4 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$	1 mk ½ mk ½ mk ½ + ½
	6.	$y = x^{cosx} + sin^{-1}\left(\frac{2x}{1 + x^2}\right)$ Let $u = x^{cosx}$ $\Longrightarrow \log u = cosx.logx$ Diff w.r.t x to get $\frac{1}{u}\frac{du}{dx} = cosx.\frac{1}{x} - logx.sinx$ $\therefore \frac{du}{dx} = x^{cosx}\left(\frac{cosx}{x} - sinx.logx\right)$ -------- (i) Let $v = sin^{-1}\left(\frac{2x}{1+x^2}\right)$ Put $x = tan\theta$ so that $v = sin^{-1}(\cos 2\theta) = 2\theta$ $\Longrightarrow v = 2\ \tan^{-1} x \quad \therefore \frac{dv}{dx} = \frac{2}{1+x^2}$ ------(ii) Now, $\frac{dy}{dx} = x^{cosx}\left(\frac{cosx}{x} - sinx.logx\right) + \frac{2}{1+x^2}$	2 mks 1 ½ mk ½ mk
	7.	(i) To show $g(x) = \|x - 5\|$ is not differentiable at $x = 5$ LHD : $\lim_{h\to 0}\frac{f(5-h)-f(5)}{-h} = \lim_{h\to 0}\frac{\|5-h-5\|-0}{-h} = -1$ RHD : $\lim_{h\to 0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to 0}\frac{\|5+h-5\|-0}{h} = 1$ Since LHD≠ RHD, $g(x) = \|x - 5\|$ is not differentiable at $x = 5$	2 mks

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		(ii) To check the continuity of function $g(x) = \|x-5\|$ at $x = 5$. LHL: $\lim_{x \to 5} (5-x) = 5-5 = 0$ RHL: $\lim_{x \to 5} (x-5) = 5-5 = 0$ And $g(5) = 0$ Since LHL= RHL, $g(x) = \|x-5\|$ is continuous at $x = 5$.	2 mks

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SET C	Q.NO	VALUE POINTS	MARK
	1.	$x = 2t^3, \quad y = \frac{3}{t^2}$ Getting $\frac{dx}{dt} = 6t^2$ and $\frac{dy}{dt} = -\frac{6}{t^3}$ Getting $\frac{dy}{dx} = -\frac{1}{t^5}$ Getting $\frac{d^2y}{dx^2} = \frac{5}{t^6} \times \frac{dt}{dx} = \frac{5}{6t^8}$	½ + ½ ½ mk ½ mk
	2.	$y = 2^{[log(x)^x]} \Longrightarrow y = 2^{x\,log x}$	½ mk

		$\Longrightarrow \log y = x\ logx. \log 2$	½ mk
		Diff w.r.t x, $\frac{1}{y}\frac{dy}{dx} = log2(1 + logx)$	½ mk
		$\therefore \frac{dy}{dx} = 2^{[log(x)^x]}\ log2(1 + logx)$	½ mk
	3.	$cos^2y + sin(xy) = c$ Diff w.r.t $x, -2\ siny. cosy. \frac{dy}{dx} + cos(xy)\left(x.\frac{dy}{dx} + y\right) = 0$ Simplifying to get $\frac{dy}{dx} = \frac{-y\ cos(xy)}{x\ cos(xy) - sin2y}$	1 mk 1 mk
	4.	$y = log\left(x + \sqrt{x^2+9}\right)$ $\frac{dy}{dx} = \frac{1}{x + \sqrt{x^2+9}}\left(1 + \frac{2x}{2\sqrt{x^2+9}}\right)$ Simplifying to get $\frac{dy}{dx} = \frac{1}{\sqrt{x^2+9}}$ Squaring and cross multiplying to get $(x^2+9)\left(\frac{dy}{dx}\right)^2 = 1$ Diff again w.r.t x and dividing by 2 $\frac{dy}{dx}$, we get $(x^2+9)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0.$	1 mk ½ mk ½ mk ½ + ½
	5.	$f(x) = \begin{cases} q - px^2, & x < 1 \\ 2x+1, & x \geq 1 \end{cases}$ is differentiable at $x = 1$. Given that f is differentiable at $x = 1 \Longrightarrow -2p(1) = 2$ $(LHD = RHD)$ Solving to get $p = -1$ Since f is differentiable, it is continuous.$\left(LHL = RHL = f(1)\right)$ $\Longrightarrow q - p = 3 \quad \therefore q = 2$	1 mk ½ mk 1 mk ½ mk
	6.		

		(i) To show $g(x) = \|x-20\|$ is not differentiable at $x = 20$ LHD : $\lim_{h\to 0} \frac{f(20-h)-f(20)}{-h} = \lim_{h\to 0} \frac{\|20-h-20\|-0}{-h} = -1$ RHD : $\lim_{h\to 0} \frac{f(20+h)-f(20)}{h} = \lim_{h\to 0} \frac{\|20+h-20\|-0}{h} = 1$ Since LHD≠ RHD, $g(x) = \|x-20\|$ is not differentiable at $x = 20$ (ii) To check the continuity of function $g(x) = \|x-20\|$ at $x = 20$. LHL: $\lim_{x\to 20}(20-x) = 20-20 = 0$ RHL: $\lim_{x\to 20}(x-20) = 20-20 = 0$ And $g(20) = 0$ Since LHL= RHL, $g(x) = \|x-20\|$ is continuous at $x = 20$.	2 mks 2 mks
	7.	$y = (tanx)^x + sec^{-1}\left(\frac{1}{2x^2-1}\right)$ Let $u = (tanx)^x$ $\Longrightarrow \log u = x \log tanx$ Diff w.r.t x to get $\frac{1}{u}\frac{du}{dx} = \frac{x}{tanx}.sec^2x + \log tanx$ $\therefore \frac{du}{dx} = (tanx)^x\left(\frac{x}{sinx\ cosx} + \log tanx\right)$ -------- (i) Let $v = sec^{-1}\left(\frac{1}{2x^2-1}\right)$ Put $x = cos\theta$ so that $v = sec^{-1}(\sec 2\theta) = 2\theta$ $\Longrightarrow v = 2\ cos^{-1}x$ $\therefore \frac{dv}{dx} = -\frac{2}{\sqrt{1-x^2}}$ ------(ii) Now, $\frac{dy}{dx} = (tanx)^x\left(\frac{x}{sinx\ cosx} + \log tanx\right) - \frac{2}{\sqrt{1-x^2}}$	2 mks 1 ½ mk ½ mk